Nominal quantification and evaluation

Standardly, natural language determiners like e.g. *no*, *most*, *at least five* are analysed as denoting functions from sets to functions from sets to truth-values (or equivalently, as relations between two sets). For example, the quantifying expression *exactly three* denotes the function which when applied to a set A results in a function which when applied to a set B is true iff there are exactly three entities in the intersection between A and B (see Peters and Westerståhl (2006) for a comprehensive exposition of generalized quantifier theory). Interestingly, no natural language determiner has been found, whose denotation DET is such that the truth of DET(A)(B) depends on the set of Bs which are not As. Put differently, it appears that DET(A)(B) holds if and only if DET(A)(A∩B) holds, i.e. the truth depends only on the sets A and A ∩ B, but not on the set B–A. Keenan and Stavi (1986) refer to this property as conservativity and hypothesise that all natural language determiners have it. But why are natural language determiners subject to this restriction? Why does no language have e.g. a determiner *exactly rouf*, such that *Exactly rouf children cry.* is true if and only if the number of crying individuals which are not children is exactly four?

Intuitively, the reason why the truth of DET(A)(B) never depends on the set of Bs which are not As is that the restrictor set does exactly what the name says, namely restrict the domain of quantification. If the domain of quantification is restricted to the elements of the set A, then the elements which are in B but not in A cannot play a role for the truth of DET(A)(B). But how can this domain-restricting function of the restrictor set be analysed? Kamp and Reyle (1993, 317) point out that conservativity follows in DRT under their assumption that the assignments verifying the scope DRS are extensions of the assignments verifying the restrictor DRS. But, arguably, this analysis is not fully satisfactory, because it answers our question only at the cost of raising another related question. Why is it that the assignments verifying the scope DRS are extensions of the assignments verifying the restrictor DRS, and not the other way around?

In this talk I argue that the reason why the truth of DET(A)(B) never depends on the set of Bs which are not in A is that determiner denotations do not make reference to a scope set B in the first place. So the basic idea is to explain the lack of determiners like *exactly rouf* by restricting the type of determiner denotations, so that denotations like that of `exactly rouf` are excluded. I reject the analysis of determiners as functions from a restrictor set into functions from a scope set into truth-values, and propose instead that they are functions from a restrictor set R into triples of the form $\langle X, C, R\rangle$, where X is a subset of R and C stands for a condition on X (and possibly R). To give some examples:
$\|no\| =_{def} \lambda R.\langle X, X \subseteq R \wedge X = \emptyset, R\rangle$
$\|most\| =_{def} \lambda R.\langle X, X \subseteq R \wedge |X| > |R - X|, R\rangle$
$\|at\ least\ three\| =_{def} \lambda R.\langle X, X \subseteq R \wedge |X| \geq 3, R\rangle$
The immediate consequence of restricting the denotation type of determiners to triples whose condition involves a subset of the restriction set (and possibly the restriction set) is that we exclude all denotations which make reference to a scope set, as well as all denotation types which make reference to the size of the domain. Since the condition in the denotation of *exactly rouf* must make reference to the scope set, this denotation is excluded by the present analysis from being a possible determiner denotation.

Assuming that the denotation of verbs are curried functions which may take triples like e.g. $\langle X, X \subseteq ST \wedge X = \emptyset, ST\rangle$ as their arguments, the semantic composition of $\|no\ student\|$ and $\|arrived\|$ results in $arrive(\langle X, X \subseteq ST \wedge X = \emptyset, ST\rangle)$. By definition of the first evaluation rule, such a formula is true if (i) there is a set X such that (ii) the conditions $X \subseteq ST \wedge X = \emptyset$ hold, and (iii) for every individual in the restrictor set ST it holds that x is in X if and only if $arrive(x)$. Since the set X is empty, and since **among the students** all and only the elements in X have the property of arriving, it follows that no student arrived. Note that in the present analysis the restrictor plays a dual function: first it provides the set from which a subset is chosen, and secondly it restricts the domain for the universal quantification in clause (iii).

More generally, a formula of the form $P(T_1, T_2, \ldots, \langle X, C(X,R), R\rangle, \ldots, T_n)$, where P is an n-place predicate, T_i are metavariables for terms (triples, atomic or non-atomic variables, individual constants), and $C(X,R)$ stands for a formula involving the set variable X (and the restrictor set R), is underspecified in various ways and can therefore be evaluated in different ways. If the NP denotations are evaluated sequentially (in an arbitrary order) this formula is true if $\exists X.C(X,R) \wedge \forall x \in R.[x \in X \leftrightarrow P(T_1, T_2, \ldots, x, \ldots, T_n)]$. So different quantifier scope relations are analysed by different orders in which the triples are evaluated. For example, after semantic composition of the two NP denotations with the verb denotation in the sentence *Two boys saw five films.*, the result is the formula $see(\langle X, X \subseteq BOY \wedge |X| = 2, BOY\rangle, \langle Y, Y \subseteq FILM \wedge |Y| = 5, FILM\rangle)$. If we evaluate the first triple first, then the result is $\exists X.X \subseteq BOY \wedge |X| = 2 \wedge \forall x \in BOY.[x \in X \leftrightarrow see(x, \langle Y, Y \subseteq FILM \wedge |Y| = 5, FILM\rangle)]$. Evaluating next the second triple, this formula is true if $\exists X.X \subseteq BOY \wedge |X| = 2 \wedge \forall x \in BOY.[x \in X \leftrightarrow \exists Y.Y \subseteq FILM \wedge |Y| = 5 \wedge \forall y \in FILM.[y \in Y \leftrightarrow see(x,y)]]$, i.e. if there are exactly two boys such that each of them saw exactly five films. On the other hand, if the triples were evaluated in the opposite order, the resulting truth-condition would be $\exists Y.Y \subseteq FILM \wedge |Y| = 5 \wedge \forall y \in FILM.[y \in Y \leftrightarrow \exists X.X \subseteq BOY \wedge |X| = 2 \wedge \forall x \in BOY.[x \in X \leftrightarrow see(x,y)]]$, i.e. there were exactly five films which were each seen by exactly two boys. Alternatively, by application of the cummulative rule the triples in the formula above can also be evaluated simultaneously. By definition of this rule, the formula is true if $\exists X.\exists Y.X \subseteq BOY \wedge |X| = 2 \wedge Y \subseteq FILM \wedge |Y| = 5 \wedge \forall x \in BOY.[x \in X \leftrightarrow \exists y \in Y.see(x,y)] \wedge \forall y \in FILM.[y \in Y \leftrightarrow \exists x \in X.see(x,y)]$.

A formula containing triples as terms can be evaluated in a number of different ways. First, the formula can be evaluated sequentially or simultaneously, to allow for quantifier scope differences as well as for cummulative readings. And secondly, each evaluation step can be either distributive (cf. universal quantification over the individuals of the restrictor set) or collective (universal quantificaton over collections of individuals in restrictor set).

In summary, if the type of determiner denotations is narrowed down to functions from restrictor sets R to triples $\langle X, C(X,R), R\rangle$ then we predict the lack of determiner denotations DET where DET(A)(B) depends on the set B–A (or the size of the domain). A relation R between such triples can be evaluated in different ways, accounting not only for scope differences, but also for cummulative, distributive and collective readings. The evaluation rules directly implement the idea that the restrictor set restricts the domain of quantification by (i) universally quantifying only over the elements of the restrictor set, and (ii) by requiring that among the individuals in the restrictor set all and only those which are in the subset X stand in the R relation to other individuals.

References

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